Hybrid Boundary Contour Mode-Matching Analysis of Arbitrarily Shaped Waveguide Structures with Symmetry of Revolution

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***Abstract*— A combined mode-matching/boundary contour mode-matching (MM/BCMM) technique is described for the rigorous and fast analysis of circular waveguide discontinuities including sections with arbitrarily shaped geometry with symmetry of revolution. The technique involves advantageously the flexibility of both the BCMM method for modeling waveguide regions of more general shape and the efficiency of the proven standard mode-matching (MM) method for circular waveguide step discontinuities. The usefulness of the hybrid method is demonstrated at the design of a spherical two-resonator filter fed by circular waveguides with circular irises. Excellent agreement with reference calculations for circular waveguide tapers verify the accuracy of the proposed method.**

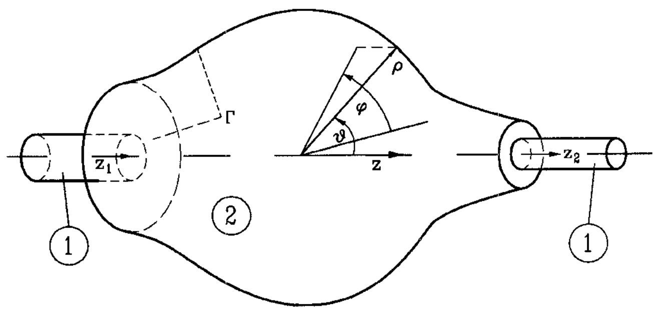

Fig. 1. Investigated rotationally symmetric mixed continuously tapered and step-type waveguide structure.

I. Introduction

CIRCULAR waveguides with more general structure in wave propagation direction have found many applications in the past for the design of continuously tapered transformers [1], horn antennas [2], [3], mode converters [4], or gyrotron cavities [5]. These structures have been investigated so far by coupled mode equations [1], [4], spherical wave expansion [2], [3], and modal analysis techniques [5]. For specific designs, such as iris-coupled spherical cavity filters, rotationally symmetric mixed continuously tapered and step-type structures (Fig. 1) are of interest. It is therefore desirable to dispose of a general, yet efficient, design method for such waveguide elements.

In this letter, the boundary contour mode-matching (BCMM) method [6], [7] is extended to include those structures. The extension is based on a hybrid mode-matching/BCMM technique with cylindrical mode expansion in the circular waveguide regions and a spherical wave expansion in the tapered sections. The example of a spherical two-resonator filter coupled by circular irises demonstrates the usefulness of the method. Comparisons with available tapered transformer results verify its accuracy.

II. Theory

The boundary contour mode-matching (BCMM) technique has already been applied successfully at arbitrarily shaped H-plane and E-plane discontinuities in rectangular waveguides [6], [7]. In the circular waveguide regions 1 (Fig. 1), the field is expanded advantageously in cylindrical wave functions, whereas for the inner region 2 with the more general boundary contour of revolution, a spherical modal expansion is adequate. The tangential field components of the TE (index h) and TM waves (index e) are formulated in both regions of the structure in Fig. 1 at the corresponding reference plane(s) by

$$\vec{E}_t^{(1)} = \sum_{j=1}^{M} \sqrt{Z_j^e}\vec{e}_j^e(a_j^e + b_j^e) + \sqrt{Z_j^h}\vec{e}_j^h(a_j^h + b_j^h) \tag{1}$$

$$\vec{H}_t^{(1)} = \sum_{j=1}^{M} \sqrt{Y_j^h}\vec{h}_j^h(\mp a_j^h \pm b_j^h) + \sqrt{Y_j^e}\vec{h}_j^e(\mp a_j^e \pm b_j^e) \tag{2}$$

$$\vec{E}_t^{(2)} = \sum_{i=1}^{N} \frac{\eta}{jk}\vec{n} \times \nabla \times \nabla \times \vec{u}_\rho \psi_i^e - \vec{n} \times \nabla \times \vec{u}_\rho \psi_i^h \tag{3}$$

$$\vec{H}_t^{(2)} = \sum_{i=1}^{N} \frac{1}{jk\eta}\vec{n} \times \nabla \times \nabla \times \vec{u}_\rho \psi_i^h + \vec{n} \times \nabla \times \vec{u}_\rho \psi_i^e \tag{4}$$

where $\vec{n}$ is the normal unity vector, Z and Y are the corresponding wave impedances or admittances, respectively, and $\vec{e}$ and $\vec{h}$ are the normal mode functions that are related to the potential functions ψ in the usual manner.

The expressions for the potential functions ψ are given by

$$\psi_{1p}^e = J_1(k_{cp}^e r) \sin\varphi (a_p^e \cdot e^{-\gamma_p z} + b_p^e \cdot e^{+\gamma_p z}) \tag{5}$$

$$\psi_{1p}^h = J_1(k_{cp}^h r) \cos\varphi (a_p^h \cdot e^{-\gamma_p z} + b_p^h \cdot e^{+\gamma_p z}) \tag{6}$$

$$\psi_{2n}^e = \alpha_n^e \hat{j}_n(k\rho) L_n^1(\cos\vartheta) \sin\varphi \tag{7}$$

$$\psi_{2n}^h = \alpha_n^h \hat{j}_n(k\rho) L_n^1(\cos\vartheta) \cos\varphi \tag{8}$$

where J_1 are the Bessel functions of the first kind, $\hat{j}_n$ are spherical *Riccati*-Bessel functions, L_n^1 are the associated Le-

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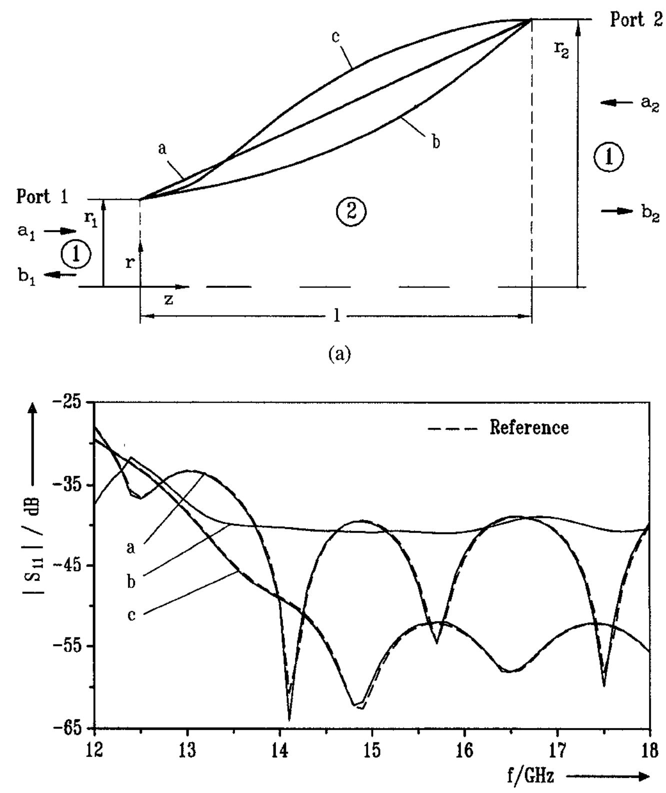

Fig. 2. Tapered circular waveguide transformers calculated with our MM/BCMM method (solid lines) and compared with results obtained by the standard mode-matching method with a step-type approximation of the taper section (by 200 steps) (dashed lines). Dimensions: $r_2 = 3r_1, r_1 = 9$ mm, $l = 70$ mm, $z_0 = 15$ mm: $d = r_2 - r_1$. a) $r(z) = r_1 + dz/l$. b) $r(z) = r_1 + dz^2/l^2$. c) $r(z) = r_1 + dz^2/(lz_0)$ $z \leq z_0$; $r(z) = r_2 + d(z-l)^2/(l(z_0-1))$ $z \geq z_0$.

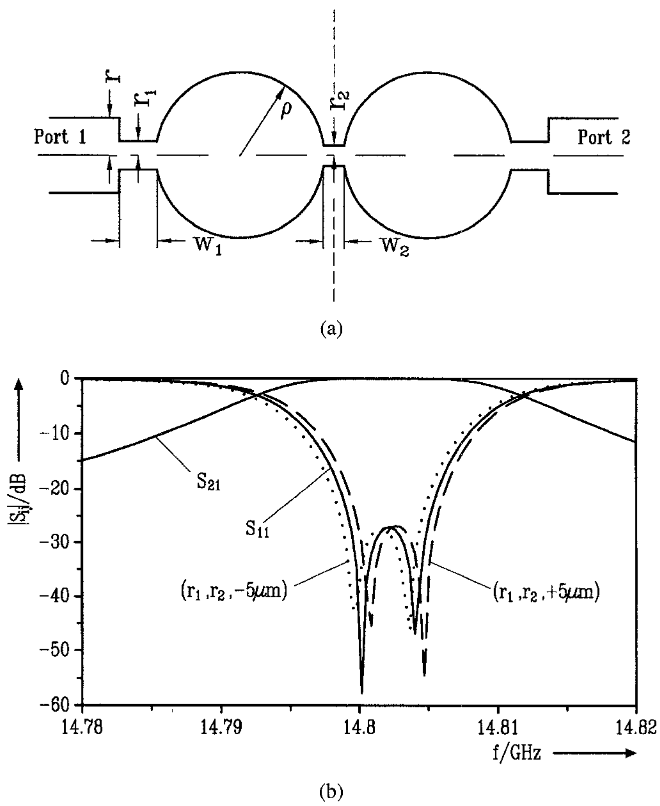

Fig. 3. S-parameters of a two-spherical cavity filter coupled by circular irises. Dimensions in mm: $r = 9, r_1 = 3, r_2 = 1.6, w_1 = 0.21, w_2 = 0.45, \rho = 15.9155$. The dashed and dotted lines in (b) show the return loss variation with a maximum tolerance of ± 5 μm for the iris diameters.

gendre functions, k_c is the cut-off wavenumber of the corresponding step discontinuity segments in region 1, and a, b, and α are the still unknown expansion coefficients.

The transition from the circular waveguide structure to the tapered region 2 is formulated by the boundary contour mode-matching method [6], [7] by enforcing the tangential electrical field strength to be zero along the conducting part and by matching the tangential electrical and magnetic field components at the common interface at the aperture.

This yields the relations

$$S\vec{\alpha} = E(\vec{a} + \vec{b}) \tag{9}$$

for the electric field, where the elements of the matrix S are the self-coupling integrals of the field vectors in region 2. Due to the nonorthogonality of these field vectors at the boundary, this matrix is not diagonal. The matrix E describes the expansion of the electric field in the horn aperture by the spherical wave functions of the inner region 2. The expressions are analogous to the cylindrical case [7]. Their rather modest complexity is demonstrated at the example of the element ee_{nm} of the matrix E

$$\begin{aligned} ee_{nm} = -q_m\pi \int_0^1 \hat{j}_n(k\rho)\{J_1(x_m u)L_n^{1\prime}(\cos\theta)\sin\theta \\ + \frac{x_m l}{\sin\theta\rho^2} J_1'(x_m u)L_n^1(\cos\theta)\}\, du \end{aligned} \tag{10}$$

with the abbreviations

$$\rho(u) = \sqrt{l^2 + (au)^2}; \quad \cos(\theta) = \frac{l}{\rho}; \quad \sin\theta = \frac{ua}{\rho} \tag{11}$$

where l is the normal distance to the waveguide port plane. The integration is carried out by Gaussian quadrature formulas with 20% oversampling concerning the oscillations.

The boundary conditions for the magnetic field at the aperture discontinuity yield the remaining sets of equations in the form

$$-\vec{a} + \vec{b} = \frac{1}{j\eta} E^T \vec{\alpha} \tag{12}$$

where η is the field impedance of region 2 and the superscript T denotes the transpose. The excitation coefficients $\vec{\alpha}$ can be eliminated by using (9), which yields

$$(-\vec{a} + \vec{b}) = C(\vec{a} + \vec{b}) \tag{13}$$

with

$$C = \frac{1}{j\eta} E^T S^{-1} E. \tag{14}$$

The desired full-wave scattering matrix T at the discontinuity is given by

$$T = (I - C)^{-1}(I + C) \tag{15}$$

with the identity matrix I. For a series of discontinuities, the generalized modal S-matrix technique is applied in the usual manner [7].

For estimating the necessary number of modes that have to be taken into account at the discontinuity, it has turned out that the most elegant and convenient way is to consider a corresponding cutoff frequency f_c up to which all propagating modes should be included. Spherical modes are propagating if the order n is less than $k_c\rho$ (for ρ see Fig. 1), where $k_c = 2\pi f_c/c$ (c is the velocity of light). The maximum order N_m for the variation of the mode functions in ϑ direction is estimated at $\rho = \rho_{\max}$ to be

$$N_m = \frac{2\pi}{c} f_c \rho_{\max} \tag{16}$$

where $\rho_{\max}$ is the maximum distance to the surface in Section II (see Fig. 1). All investigations of the convergence behavior can then be carried out in a convenient manner by simply increasing f_c for a given operating frequency f_0, where for most applications $f_c = 2, \cdots, 15 f_0$ is a suitable range.

III. Results

For reference values, tapered circular waveguide transformers [1] are calculated (Fig. 2) with our method and compared with results obtained by the standard mode-matching method with a step-type approximation of the taper section (by 200 steps), which is significantly more CPU intensive. Twenty and 16 eigenmodes at the taper input and output ports, respectively, have been used for the reference calculations. This choice would correspond to a maximum cutoff frequency of about $f_c = 180$ GHz. In our method, merely $f_c = 50$ GHz is required to yield convergent results. The total CPU time for the complete analysis of, e.g., the linear profile taper is only 30 CPUsec at a 486/33 PC, in comparison with 650 CPUsec (on the same PC) required for an approximation by 200 steps (and utilizing the fast mode-matching technique of, e.g., [8]). This may demonstrate the high computational efficiency of the presented hybrid method.

To prove the flexibility of the method, in Fig. 3, the S-parameters are presented of a two-spherical cavity filter coupled by circular irises. In order to yield the required numerical convergence for this example, it has turned out that merely all modes up to the cutoff frequency of $f_c = 200$ GHz are necessary (i.e., up to only four times the cutoff frequency that was required for the linear taper). The filter structure is rather insensitive to tolerances concerning the iris diameters r_1, r_2. This is demonstrated in Fig. 3(b) (dashed and dotted lines) by including the maximum tolerance of ± 5 μm. Please note the very enlarged scale. As a broader frequency range is not shown in Fig. 3(b), the calculated insertion loss values of about 40 dB for 14.7 and 14.9 GHz may demonstrate the steepness of the filter.

IV. Conclusion

A very powerful hybrid MM/BCMM technique is presented for the rigorous analysis of mixed circular waveguide structures including step-type discontinuities and arbitrarily shaped geometries with symmetry of revolution. The technique combines the advantages of both the efficiency of the standard mode-matching method and the flexibility of the BCMM technique. A spherical cavity filter example demonstrates the usefulness of the MM/BCMM CAD method.

Acknowledgment

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